Online Supplemental Materials

Summary of Deficient Rank Factor Loading Conditions

Condition	Trait Loadings	Method Loadings	Residual Variance	Factor *r*	*N*
1, 28, 55	.3, .3, .3	.90, .90, .90	0.10	0.20	50, 150, 300
2, 29, 56	.7, .7, .7	.64, .64, .64	0.10	0.20	50, 150, 300
3, 30, 57	.3, .3, .7	.90, .90, .64	0.10	0.20	50, 150, 300
4, 31, 58	.3, .3, .3	.84, .84, .84	0.20	0.20	50, 150, 300
5, 32, 59	.7, .7, .7	.56, .56, .56	0.20	0.20	50, 150, 300
6, 33, 60	.3, .3, .7	.84, .84, .56	0.20	0.20	50, 150, 300
7, 34, 61	.3, .3, .3	.71, .71, .71	0.40	0.20	50, 150, 300
8, 35, 62	.7, .7, .7	.33, .33, .33	0.40	0.20	50, 150, 300
9, 36, 63	.3, .3, .7	.71, .71, .33	0.40	0.20	50, 150, 300
10, 37, 64	.3, .3, .3	.90, .90, .90	0.10	0.50	50, 150, 300
11, 38, 65	.7, .7, .7	.64, .64, .64	0.10	0.50	50, 150, 300
12, 39, 66	.3, .3, .7	.90, .90, .64	0.10	0.50	50, 150, 300
13, 40, 67	.3, .3, .3	.84, .84, .84	0.20	0.50	50, 150, 300
14, 41, 68	.7, .7, .7	.56, .56, .56	0.20	0.50	50, 150, 300
15, 42, 69	.3, .3, .7	.84, .84, .56	0.20	0.50	50, 150, 300
16, 43, 70	.3, .3, .3	.71, .71, .71	0.40	0.50	50, 150, 300
17, 44, 71	.7, .7, .7	.33, .33, .33	0.40	0.50	50, 150, 300
18, 45, 72	.3, .3, .7	.71, .71, .33	0.40	0.50	50, 150, 300
19, 46, 73	.3, .3, .3	.90, .90, .90	0.10	0.80	50, 150, 300
20, 47, 74	.7, .7, .7	.64, .64, .64	0.10	0.80	50, 150, 300
21, 48, 75	.3, .3, .7	.90, .90, .64	0.10	0.80	50, 150, 300
22, 49, 76	.3, .3, .3	.84, .84, .84	0.20	0.80	50, 150, 300
23, 50, 77	.7, .7, .7	.56, .56, .56	0.20	0.80	50, 150, 300
24, 51, 78	.3, .3, .7	.84, .84, .56	0.20	0.80	50, 150, 300
25, 52, 79	.3, .3, .3	.71, .71, .71	0.40	0.80	50, 150, 300
26, 53, 80	.7, .7, .7	.33, .33, .33	0.40	0.80	50, 150, 300
27, 54, 81	.3, .3, .7	.71, .71, .33	0.40	0.80	50, 150, 300

Note: Each triplet of condition numbers corresponds to each triplet of sample sizes. In contrast, the triplets for factor loadings are identical across each of the three conditions. For example, the 1st, 28th, and 55th conditions have the same trait loadings, residual variances, and inter-factor correlations, but have different samples sizes.

Incorporating Prior Distributions into Estimation of the CT-CM Model

This section aims to illustrate the inclusion of prior distributions, and stimulate future research that identifies best practices for selection of prior distributions. More specifically, this section provides a method for identifying prior distributions for certain parameters within the CT-CM model, describes the assumptions/limitations of the method, demonstrates the method by extending the one of the empirical examples from the main text (PAIRS data), and concludes with future directions for quantitative researchers interested in priors distributions for the CT-CM model.

Informative Prior Distributions

Informative prior distributions represent a researcher's knowledge/belief of a specific parameter before data collection. For the CT-CM model, researchers may have prior knowledge of factor loadings, inter-factor correlations, manifest variable means, or residual variances. That knowledge may be articulated as a distribution, and then included within a Bayesian analysis. For example, a researcher may form an informative normal prior for a factor loading based on the mean and standard deviation of factor loadings reported within similar research studies (i.e. comparable manifest variables fitted to a comparable model). The remainder of this exposition exemplifies this case by describing how to gather information based on previous research, and then use the information to identify informative priors.

Gathering Information for Prior Distributions

A researcher that aims to define an informative prior based on previous literature would, ideally, gather information from previous studies that analyze data reflecting the same constructs, from the same reference population, using the same model. Naturally, the ideal case rarely occurs in practice; studies often use different constructs, models, and reference populations.

Therefore, researchers must select comparable studies, and report relevant discrepancies between the current examination the previous research.

To augment our first empirical example, we sought previous research that examined self-, parent-, or peer-reports of personality using an MTMM confirmatory factor model, and found six studies (Barbaranelli et al., 2008; Hong et al., 2008; Biesanz & West, 2004; Baker et al., 2004; Lim & Ployhart, 2006; DeYoung, 2006). Yet, we note that some of these studies differed from our first empirical example based on either the MTMM model fitted to data, the type of reporter (we originally had self, friend, and parent repors) used within the study, or the reference population for the participants. None of the previous studies used the CT-CM model; four fitted the CT-CU model and two fitted the CT-UM. Three studies contained only self-reports; one had both self- and peer-reports; one had self-, parent- and teacher-reports; and one had self-, teacher-, and peer-reports. One study examined adults ($N = 490$), one examined emerging adolescents ($N = 391$), one examined adolescents ($N = 165$), and three examined undergraduates ($N = 295$; $N = 387$; $N = 353$). Hence, the previous studies largely matched the focus of our second empirical example (i.e. they measured the same constructs and performed an MTMM analysis), and we assume negligible differences between our second empirical example and the previous studies based on the specific MTMM model fitted, the type of reporter used, or the reference population.

Identifying Prior Distributions for Factor Loadings

Next, we describe how to transform factor loadings from previous research into informative prior distributions. We narrowed our focus to standardized trait factor loadings because standardized factor loadings are comparable across studies, and most of the prior studies did not estimate method factors. Specifically, we extracted 65 standardized factor loadings for the agreeableness, conscientiousness, and openness to experience factors. Next, we aimed to summarize the standardized factor loadings as a distribution (i.e. form an informative prior distribution) via examination of histograms and summary statistics. Figure S-1 contains the histograms, which separates self-, peer-, and parent-/teacher-reported loadings for conscientiousness, agreeableness, and openness to experience. We combined estimates of parent and teacher loadings because we only identified three factor loadings for parent-reported manifest variables, and we recognize that this combination may be less justified from a theoretical perspective. Within Figure S-1, rows refer to personality traits (1^{st} row is conscientiousness, 2^{nd} row is agreeableness, 3^{rd} row is openness to experience) and columns indicate reporter type (1^{st} column is self-reports, 2^{nd} column is peer-reports, and 3^{rd} columns is parent/teacher reports). Table S-1 summarizes the mean, min, and max factor loading for each reporter type and personality trait. The summary statistics and histograms indicate that the factors loadings tend to be positive (i.e., all occur within [.24, .92]), and have an average of .66. Accordingly, we translate this information into a normal distribution with a .65 mean and a .30 standard deviation. More specifically, the prior mean of .65 closely matches the mean across all of the cells of Figure S-1, and the prior standard deviation of .30 produces probability mass for positive loadings rather than negative loadings (i.e., 97.5% of standardized factor loadings are larger than .05).

For application, the informative prior distribution for the factor loadings must be converted from the standardized metric to a raw metric. A standardized loading (generically labeled $\lambda^*_{F,Y}$ for factor F and manifest variable Y), equals the raw loading, $\lambda_{F,Y}$, multiplied by the the standard deviation of the factor, s_F, and divided by the standard deviation of the manifest variable, s_Y (i.e. $\lambda^*_{F,Y} = \lambda_{F,Y}\left(\frac{s_F}{s_Y}\right)$). Given that the standard deviation of the factor equals 1 (i.e. we identify the CT-CM model by constraining the factor variances to unity), the raw loading may be obtained by multiplying by the standard deviation of the manifest variable (i.e. $\lambda_{F,Y} = \lambda^*_{F,Y} s_Y$). Accordingly, we may transform the prior distribution from the standardized metric to the raw metric by multiplying both the prior mean and standard deviation by each manifest variable's standard deviation.

Identifying Prior Distributions for Latent Variable Correlations

Next, we describe how to identify an informative prior distribution for the latent variable correlations. Analogous to the identification of informative prior distributions for the manifest variable loadings, we began by extracting 21 latent variable correlations from the six focal empirical reports. Next, we calculated summary statistics and created histograms of the correlations (see Table S-1 and Figure S-2, respectively). When calculating means of the correlations, we first transformed each correlation to Fisher's *z*-metric, then averaged correlations in *z*-metric, and finally transformed the averages back to the correlation metric. Generally, the summary information from Table S-1 and Figure S-2 indicates that the correlations are centered around .3 and tend to be positive (i.e., remain within [-.13, .67]). Hence, we aim to identify a prior distribution with mainly positive correlations.

We implemented an inverse-Wishart prior distribution for the latent variable variance-covariance matrices (see Equation 9 in the main text), which requires a reference variance-covariance matrix $\mathbf{\Omega}$, and degrees of freedom d. Thus, we can make the inverse-Wishart prior informative by changing either $\mathbf{\Omega}$ or d, and we aim to identify values for $\mathbf{\Omega}$ or d that match positive correlations centered about .30. We simulated data from an inverse-Wishart distribution with off-diagonal elements of $\mathbf{\Omega}$ equal to .1, .2, and .3, and degrees of freedom equal to 5, 7, 10, and 20. Figure S-3 contains the densities of these distributions, with separate plots for the different off diagonal values of $\mathbf{\Omega}$ (generically labeled as $\mathbf{\Omega}_{ij}$), different line types corresponding to different degrees of freedom, and a solid vertical line denoting the location of a .30 correlation. Inspection of Figure S-3 shows that an inverse-Wishart distribution with off diagonal elements equal to .2 with degrees of freedom equal to 7 matches our original notion. In particular, this distribution is centered around a correlation of .30, and has density mass for mainly positive values. Therefore, we adopt an inverse-Wishart distribution with **Ω** equal to 1 along the main diagonal and .2 in the off-diagonal, and 7 degrees of freedom.

Comparison of Minimally Informative Priors and Informative Priors

Next, we demonstrate the difference of the results for minimally informative priors versus informative priors. In particular, reanalyze the personality data from our first empirical example using the prior distributions developed within the previous two subsections. The results are summarized within Table S-2, with results using minimally informative priors under the column labeled 'Min. Info', and those results from informative priors under the column labeled 'Inf. Prior'. Visual inspection of Table S-2 shows that the results from the different approaches were essentially indistinguishable. Therefore, the inclusion of the prior distributions, as outlined within the previous two sections, had little effect on the parameter estimates for the CT-CM model fitted to the PAIRS data.

Conclusions and Future Directions

This section of the online supplemental material identified a method for defining prior distributions for parameters within the CT-CM model. The method included identifying comparable studies, extracting standardized factor loadings and correlations from those studies, and summarizing the estimates as a distribution (via examination of histograms and summary statistics). Although our demonstration of this method had negligible effects on the results (as compared to minimally informative priors), we expect a greater impact for stronger prior distributions (i.e. normal distributions with smaller standard deviations; inverse-Wishart distributions with larger degrees of freedom), empirical examples with smaller sample sizes, or both. Importantly, researchers should provide rationale for the use of informative priors (e.g., histograms and summary statistics of the previous parameter estimates), and include the assumptions embedded within the formation of the priors (e.g., discrepancies between the statistical models or reference populations).

This section also illuminates two pathways for future research. First, future work should examine the appropriateness of forming prior distributions from different MTMM models. It is unlikely that previous research would provide CT-CM parameter estimates given its history of estimation problems. Hence, we gleaned information from alternative CT-CM models that accounted for method sources of variance (e.g. CT-CU, CT-UM), and therefore should produce comparable trait factor loadings and correlations for the CT-CM model. Yet, this assumption could be verified (or rejected) via simulation. Second, a future study should identify the minimal strength needed for a prior distribution to impact results. Our empirical demonstration showed negligible effects of our selected informative priors distributions (compared to minimally informative priors), even though we selected the empirical example with the smallest sample size. A thorough investigation that examines the impact of a prior given sample size, model complexity, the strength of a particular effect would help elucidate which applications would benefit most from the inclusion of informative priors.

Overall, this section offers one method for developing informative prior distributions for the application of the CT-CM model to MTMM data. We hope this guide will aid those seeking to include informative priors within their own analyses, and stimulate future work on forming best practices for defining informative priors.

Table S-1. Summary of Parameter Estimates from Previous Research

Parameter	*Reporter*	*Trait*	*No. of Est.*	*Mean*	*Min*	*Max*
Loading						
	Parent/Teacher	A	3	0.58	0.31	0.76
	Parent/Teacher	C	3	0.70	0.47	0.83
	Parent/Teacher	O	3	0.73	0.46	0.89
	Peer	A	7	0.53	0.46	0.60
	Peer	C	7	0.62	0.53	0.78
	Peer	O	7	0.64	0.54	0.85
	Self	A	12	0.64	0.24	0.88
	Self	C	12	0.66	0.39	0.91
	Self	O	11	0.68	0.25	0.92
Correlation						
	--	A with C	7	0.28	-0.13	0.58
	--	A with O	7	0.34	-0.04	0.67
	--	C with O	7	0.34	-0.16	0.80

Note: ‘A’ refers to agreeableness, ‘C’ to conscientiousness, and ‘O’ to openness to experience. ‘No. of Est.’ refers to the number of estimates used to create the summary statistics.

Table S-2. Bayesian Parameter Estimates with Minimally Informative and Informative Priors for the PAIRS Data

Parameter	Min. Info.	ML 95% CI	Info. Prior	Bayes 95% CI
$\lambda_{T1,Y1}$	1.67	[1.13, 2.12]	1.61	[1.14 2.05]
$\lambda_{T1,Y4}$	1.32	[0.84, 1.84]	1.35	[0.90 1.84]
$\lambda_{T1,Y7}$	0.86	[0.47, 1.28]	0.90	[0.53 1.28]
$\lambda_{T2,Y2}$	1.99	[1.56, 2.41]	1.94	[1.55 2.35]
$\lambda_{T2,Y5}$	1.62	[1.17, 2.08]	1.60	[1.20 2.04]
$\lambda_{T2,Y8}$	1.32	[0.88, 1.78]	1.32	[0.91 1.75]
$\lambda_{T3,Y3}$	1.90	[1.43, 2.31]	1.85	[1.41 2.24]
$\lambda_{T3,Y6}$	1.13	[0.74, 1.55]	1.15	[0.79 1.54]
$\lambda_{T3,Y9}$	1.05	[0.69, 1.46]	1.07	[0.74 1.44]
$\lambda_{M1,Y1}$	1.02	[0.30, 1.61]	1.07	[0.39 1.63]
$\lambda_{M1,Y2}$	0.98	[0.29, 1.51]	1.00	[0.36 1.52]
$\lambda_{M1,Y3}$	0.34	[-0.32, 1.02]	0.32	[-0.28 0.95]
$\lambda_{M2,Y4}$	1.34	[0.84, 1.89]	1.33	[0.83 1.87]
$\lambda_{M2,Y5}$	1.29	[0.77, 1.79]	1.28	[0.77 1.78]
$\lambda_{M2,Y6}$	1.40	[0.94, 1.87]	1.41	[0.94 1.88]
$\lambda_{M3,Y7}$	0.41	[0.01, 0.85]	0.41	[0.01 0.84]
$\lambda_{M3,Y8}$	1.33	[0.61, 2.02]	1.30	[0.58 1.99]
$\lambda_{M3,Y9}$	1.00	[0.53, 1.61]	1.01	[0.54 1.64]
$\sigma^2_{RV_{Y1}}$	0.51	[0.02, 1.74]	0.57	[0.03 1.80]
$\sigma^2_{RV_{Y2}}$	0.73	[0.06, 1.97]	0.78	[0.06 2.01]
$\sigma^2_{RV_{Y3}}$	0.96	[0.05, 2.28]	1.09	[0.11 2.38]
$\sigma^2_{RV_{Y4}}$	2.56	[1.34, 3.90]	2.53	[1.30 3.83]
$\sigma^2_{RV_{Y5}}$	2.17	[1.02, 3.34]	2.18	[1.03 3.40]
$\sigma^2_{RV_{Y6}}$	1.55	[0.39, 2.63]	1.47	[0.29 2.62]
$\sigma^2_{RV_{Y7}}$	2.56	[1.83, 3.47]	2.52	[1.81 3.44]
$\sigma^2_{RV_{Y8}}$	1.87	[0.13, 3.34]	1.91	[0.20 3.33]
$\sigma^2_{RV_{Y9}}$	1.70	[0.33, 2.71]	1.65	[0.25 2.67]
$\rho_{T1,T2}$	0.13	[-0.18, 0.37]	0.11	-0.17 0.35
$\rho_{T1,T3}$	-0.04	[-0.36, 0.20]	-0.04	-0.33 0.20
$\rho_{T2,T3}$	-0.28	[-0.53, -0.03]	-0.26	-0.49 -0.02
$\rho_{M1,M2}$	-0.02	[-0.46, 0.34]	-0.01	-0.40 0.32
$\rho_{M1,M3}$	0.01	[-0.43, 0.37]	0.01	-0.38 0.34
$\rho_{M2,M3}$	0.08	[-0.24, 0.36]	0.07	-0.23 0.35

Note: Traits 1, 2, and 3 refer to agreeableness, conscientiousness, and openness to experience, respectively. Methods 1, 2, and 3 refer to self, friend, and parent report, respectively. Column labeled ‘Min. Info.’ contains results from minimally informative priors, and ‘Info. Prior’ has results using informative priors.

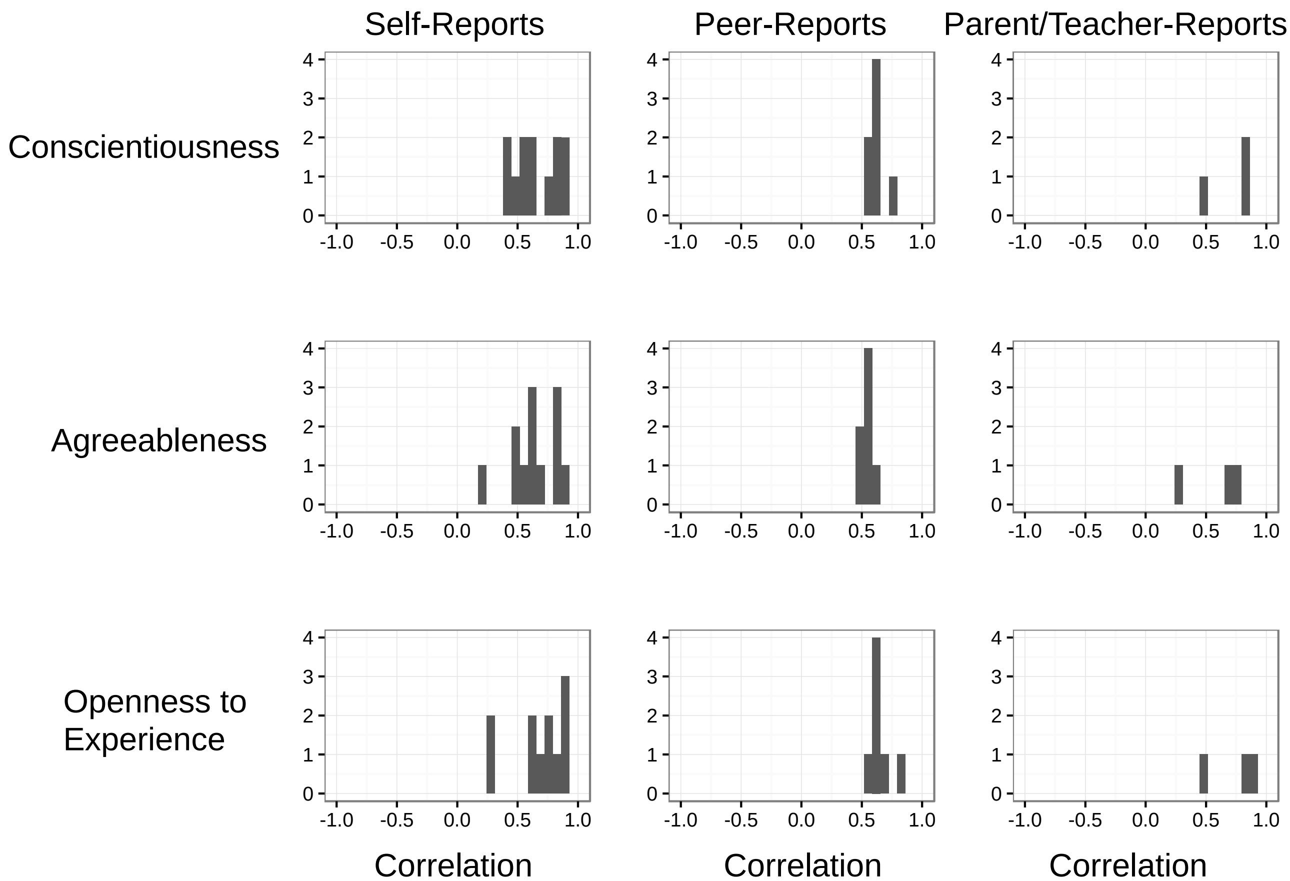

Figure S-1. Histograms of standardized factor loadings from six previous studies of personality.

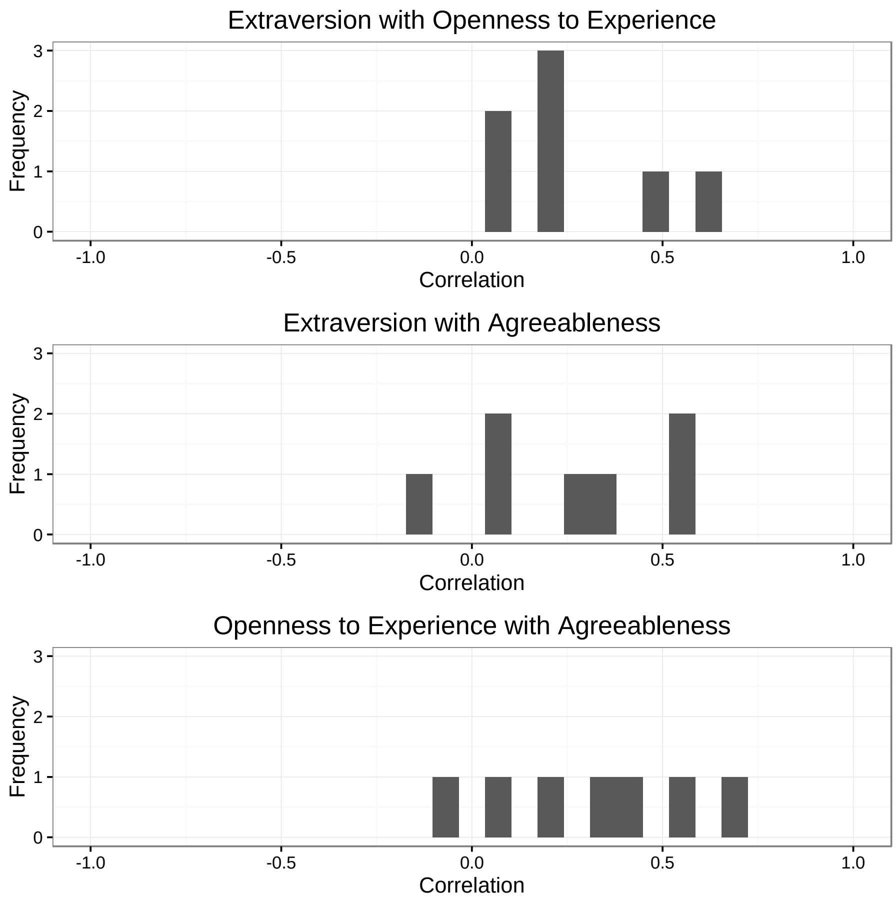

Figure S-2. Histograms of correlations between latent factors of personality from six previous studies.

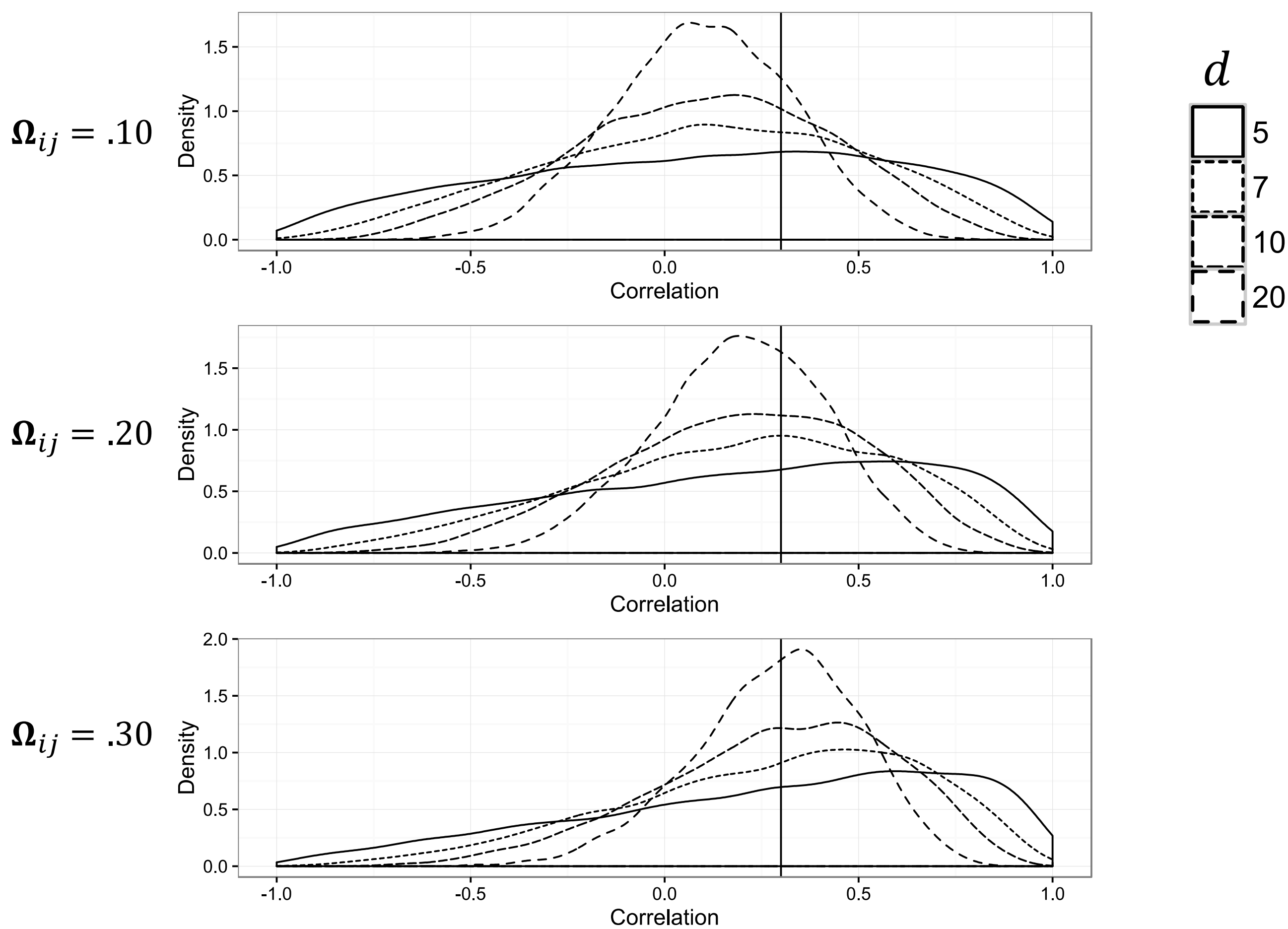

Figure S-3. Simulated densities for the inverse-Wishart distribution with different off diagonal values (labeled $\mathbf{\Omega}_{ij}$) and degrees of freedom (labeled d). The vertical bar designates the locations of a .30 correlation.